2.1 Projectile Motion

SECTION EXPECTATIONS

- Describe qualitatively and quantitatively the path of a projectile.
- Analyze, predict, and explain projectile motion in terms of horizontal and vertical components.
- Design and conduct experiments to test the predictions about the motion of a projectile.

KEY TERMS

- trajectory
- projectile
- range
- parabola

Figure 2.1 After the water leaves the pipes in this fountain, the only forces acting on the water are gravity and air friction.

A tourist visiting Monte Carlo, Monaco, would probably stand and admire the beauty of the fountain shown in the photograph, and might even toss a coin into the fountain and make a wish. A physics student, however, might admire the symmetry of the water jets. He or she might estimate the highest point that the water reaches and the angle at which it leaves the fountain, and then mentally calculate the initial velocity the water must have in order to reach that height.

The student might then try to think of as many examples of this type of motion as possible. For example, a golf ball hit off the tee, a leaping frog, a punted football, and a show-jumping horse all follow the same type of path or **trajectory** as the water from a fountain. Any object given an initial thrust and then allowed to soar through the air under the force of gravity only is called a **projectile**. The horizontal distance that the projectile travels is called its **range**.

Air friction does, of course, affect the trajectory of a projectile and therefore the range of the projectile, but the mathematics needed to account for air friction is complex. You can learn a great deal about the trajectory of projectiles by neglecting friction, while keeping in mind that air friction will modify the actual motion.

You do not need to learn any new concepts in order to analyze and predict the motion of projectiles. All you need are data that will provide you with the velocity of the projectile at the moment it is launched and the kinematic equations for uniformly accelerated motion. You observed projectile motion in the Race

HISTORY LINK

Calculating actual trajectories of artillery shells was an enormous task before the advent of electronic computers. Human experts required up to 20 h to do the job, even with mechanical calculators. ENIAC, the first electronic computer that stored its program instructions, was built in 1946 and could calculate the trajectory of an artillery shell in about 30 s. The vacuum tubes in ENIAC's processing unit required 174 kW of electric power, so the energy required to calculate one trajectory was comparable to the energy needed to actually fire the artillery shell!

to the Ground segment of the Multi-Lab and identified a feature of the motion that simplifies the analysis. The horizontal motion of the projectile does not influence the vertical motion, nor does the vertical motion affect the horizontal motion. *You can treat the motion in the two directions independently.* The following points will help you analyze all instances of projectile motion.

- Gravity is the only force influencing ideal projectile motion. (Neglect air friction.)
- Gravity affects only the vertical motion, so equations for uniformly accelerated motion apply.
- No forces affect horizontal motion, so equations for uniform motion apply.
- The horizontal and vertical motions are taking place during the same time interval, thus providing a link between the motion in these dimensions.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of two-dimensional motion, go to your Electronic Learning Partner for an interactive activity.

Projectiles Launched Horizontally

If you had taken a picture with a strobe light of your Race to the Ground lab, you would have obtained a photograph similar to the one in Figure 2.2. The ball on the right was given an initial horizontal velocity while, at the same moment, the ball on the left was dropped. As you can see in the photograph, the two balls were the same distance from the floor at any given time — the vertical motion of the two balls was identical. This observation verifies that horizontal motion does not influence vertical motion. Examine the following sample problem to learn how to make use of this feature of projectile motion.

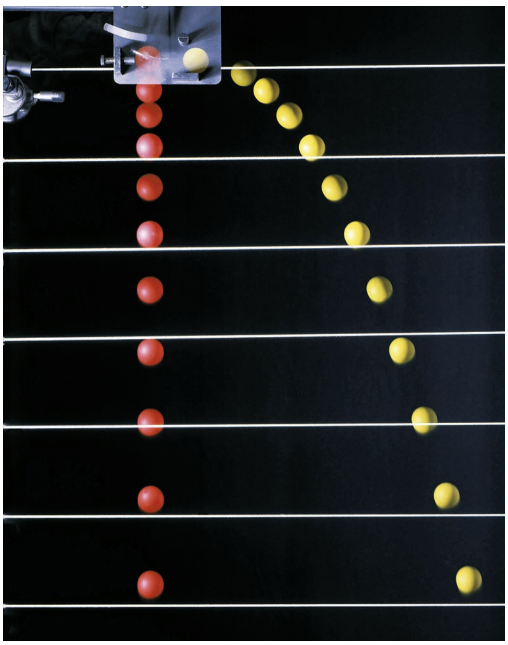


Figure 2.2 You can see that the balls are accelerating downward, because the distances they have travelled between flashes of the strobe light are increasing. If you inspected the horizontal motion of the ball on the right, you would find that it travelled the same horizontal distance between each flash of the strobe light.

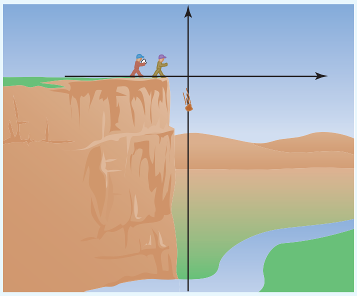
SAMPLE PROBLEM

Analyzing a Horizontal Projectile

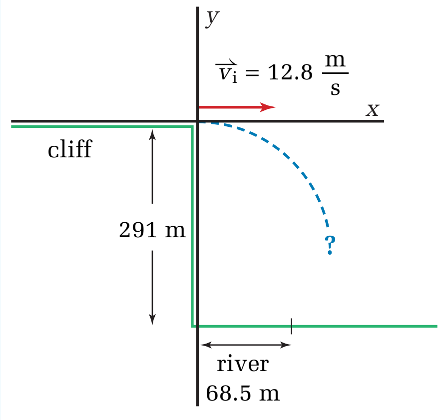
While hiking in the wilderness, you come to a cliff overlooking a river. A topographical map shows that the cliff is 291 m high and the river is 68.5 m wide at that point. You throw a rock directly forward from the top of the cliff, giving the rock a horizontal velocity of 12.8 m/s.

(a) Did the rock make it across the river?

(b) With what velocity did the rock hit the ground or water?



Conceptualize the Problem



- Start to frame the problem by making a rough sketch of the cliff with a *coordinate system* superimposed on it. Write the initial conditions on the sketch.
- The rock *initially* has *no vertical velocity*. It falls, from *rest*, with the *acceleration due to gravity*. Since "down" was chosen as negative, the *acceleration* of the rock is *negative*. (Neglect air friction.)
- Since the coordinate system was placed at the top of the cliff, the *vertical component* of the *displacement* of the rock is *negative*.
- The *displacement* that the rock falls determines the *time interval* during which it falls, according to the kinematic equations.
- The rock moves *horizontally* with a *constant velocity* until it hits the ground or water at the end of the time interval.
- The *final velocity* of the rock at the instant before it hits the ground or water is the *vector sum* of the horizontal velocity and the final vertical velocity.
- Use x to represent the horizontal component of displacement and y for the vertical component of displacement. Use x and y subscripts to identify the horizontal and vertical components of the velocity.

Identify the Goal

(a) Whether the horizontal distance, Δx, travelled by the rock was greater than 68.5 m, the width of the river

(b) The final velocity, $\vec{v}_f$, of the rock the instant before it hit the ground

Identify the Variables

Known		Implied	Unknown
$\Delta y = -291$ m	river width = 68.5 m	$a_y = -9.81\ \frac{m}{s^2}$	Δx
$v_x = 12.8\ \frac{m}{s}$		$v_{iy} = 0.0\ \frac{m}{s}$	$\vec{v}_f$

Develop a Strategy

Find the time interval during which the rock was falling by using the kinematic equation that relates displacement, initial velocity, acceleration, and time interval. Note that the vertical component of the initial velocity is zero and solve for the time interval.

$$\Delta y = v_{yi}\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta y = \frac{1}{2}a\Delta t^2$$

$$\frac{2\Delta y}{a} = \Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Insert numerical values and solve.

$$\Delta t = \sqrt{\frac{2(-291\ \cancel{\text{m}})}{-9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}}$$

$$\Delta t = \sqrt{59.327\ \text{s}^2}$$

$$\Delta t = \pm 7.7024\ \text{s}$$

Find the horizontal displacement of the rock by using the equation for uniform motion (constant velocity) that relates velocity, distance, and time interval. Solve for displacement.

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x \Delta t$$

Use the time calculated above and initial velocity to calculate the horizontal distance travelled by the rock. Choose the positive value for time, since negative time has no meaning in this application.

$$\Delta x = \left(12.8\ \frac{\text{m}}{\cancel{\text{s}}}\right)(7.7024\ \cancel{\text{s}})$$

$$\Delta x = 98.591\ \text{m}$$

$$\Delta x \cong 98.6\ \text{m}$$

(a) Since the horizontal distance travelled by the rock (98.6 m) was much greater than the width of the river (68.5 m), the rock hit the ground on the far side of the river.

Find the vertical component of the final velocity by using the kinematic equation that relates initial velocity, final velocity, acceleration, and time.

$$v_{fy} = v_{iy} + a\Delta t$$

Insert the numerical values and solve.

$$v_{fy} = 0.0\ \frac{\text{m}}{\text{s}} + \left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(7.7024\ \text{s})$$

$$v_{fy} = -75.561\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity.

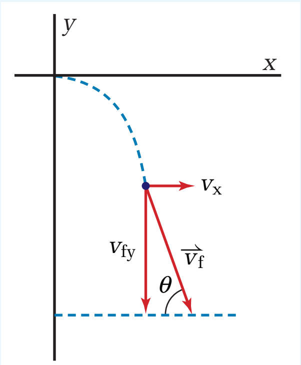
$$|\vec{v}_f| = \sqrt{(v_x)^2 + (v_{fy})^2}$$

$$|\vec{v}_f| = \sqrt{\left(12.8\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-75.561\ \frac{\text{m}}{\text{s}}\right)^2}$$

$$|\vec{v}_f| = \sqrt{5873.30\ \frac{\text{m}^2}{\text{s}^2}}$$

$$|\vec{v}_f| = 76.637\ \frac{\text{m}}{\text{s}}$$

$$|\vec{v}_f| \cong 76.6\ \frac{\text{m}}{\text{s}}$$



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Use trigonometry to find the angle that the rock made with the horizontal when it struck the ground.

$$\tan\theta = \frac{v_{fy}}{v_x}$$

$$\theta = \tan^{-1}\frac{v_{fy}}{v_x}$$

$$\theta = \tan^{-1}\frac{75.561\ \frac{m}{s}}{12.8\ \frac{m}{s}}$$

$$\theta = \tan^{-1} 5.9032$$

$$\theta = 80.385^\circ$$

$$\theta \cong 80.4^\circ$$

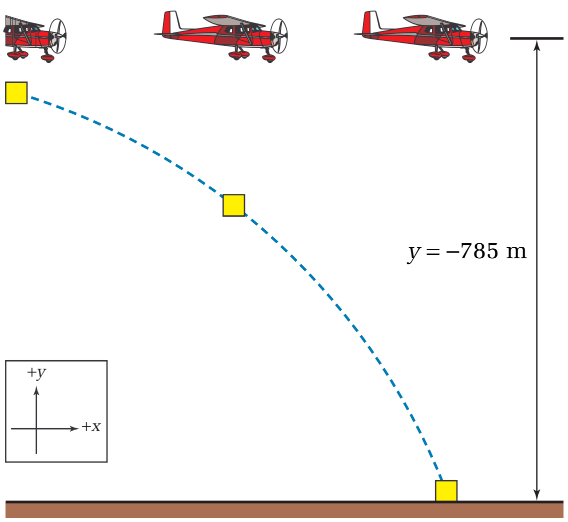
(b) The rock hit the ground with a velocity of 76.6 m/s at an angle of 80.4° with the horizontal.

Validate the Solution

The distance that the rock fell vertically was very large, so you would expect that the rock would be travelling very fast and that it would hit the ground at an angle that was nearly perpendicular to the ground. Both conditions were observed.

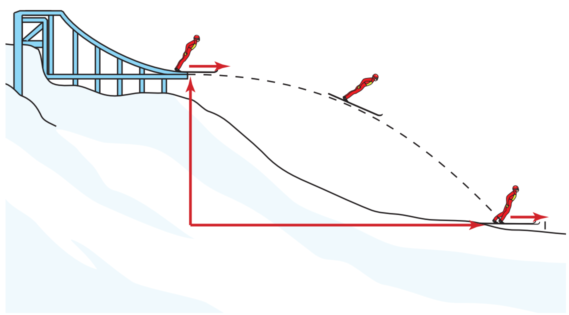
PRACTICE PROBLEMS

1. An airplane is dropping supplies to northern villages that are isolated by severe blizzards and cannot be reached by land vehicles. The airplane is flying at an altitude of 785 m and at a constant horizontal velocity of 53.5 m/s. At what horizontal distance before the drop point should the co-pilot drop the supplies so that they will land at the drop point? (Neglect air friction.)



2. A cougar is crouched on the branch of a tree that is 3.82 m above the ground. He sees an unsuspecting rabbit on the ground, sitting 4.12 m from the spot directly below the branch on which he is crouched. At what horizontal velocity should the cougar jump from the branch in order to land at the point at which the rabbit is sitting?

3. A skier leaves a jump with a horizontal velocity of 22.4 m/s. If the landing point is 78.5 m lower than the end of the ski jump, what horizontal distance did the skier jump? What was the skier's velocity when she landed? (Neglect air friction.)



4. An archer shoots an arrow toward a target, giving it a horizontal velocity of 70.1 m/s. If the target is 12.5 m away from the archer, at what vertical distance below the point of release will the arrow hit the target? (Neglect air friction.)

5. In a physics experiment, you are rolling a golf ball off a table. If the tabletop is 1.22 m above the floor and the golf ball hits the floor 1.52 m horizontally from the table, what was the initial velocity of the golf ball?

6. As you sit at your desk at home, your favourite autographed baseball rolls across a shelf at 1.0 m/s and falls 1.5 m to the floor. How far does it land from the base of the shelf?

7. A stone is thrown horizontally at 22 m/s from a canyon wall that is 55 m high. At what distance from the base of the canyon wall will the stone land?

8. A sharpshooter shoots a bullet horizontally over level ground with a velocity of 3.00×10^2 m/s. At the instant that the bullet leaves the barrel, its empty shell casing falls vertically and strikes the ground with a vertical velocity of 5.00 m/s.
   (a) How far does the bullet travel?
   (b) What is the vertical component of the bullet's velocity at the instant before it hits the ground?

Projectiles Launched at an Angle

Most projectiles, including living ones such as the playful dolphins in Figure 2.3, do not start their trajectory horizontally. Most projectiles, from footballs to frogs, start at an angle with the horizontal. Consequently, they have an initial velocity in both the horizontal and vertical directions. These trajectories are described mathematically as **parabolas**. The only additional step required to analyze the motion of projectiles launched at an angle is to determine the magnitude of the horizontal and vertical components of the initial velocity.

Mathematically, the path of any ideal projectile lies along a parabola. In the following investigation, you will develop some mathematical relationships that describe parabolas. Then, the sample problems that follow will help you apply mathematical techniques for analyzing projectiles.

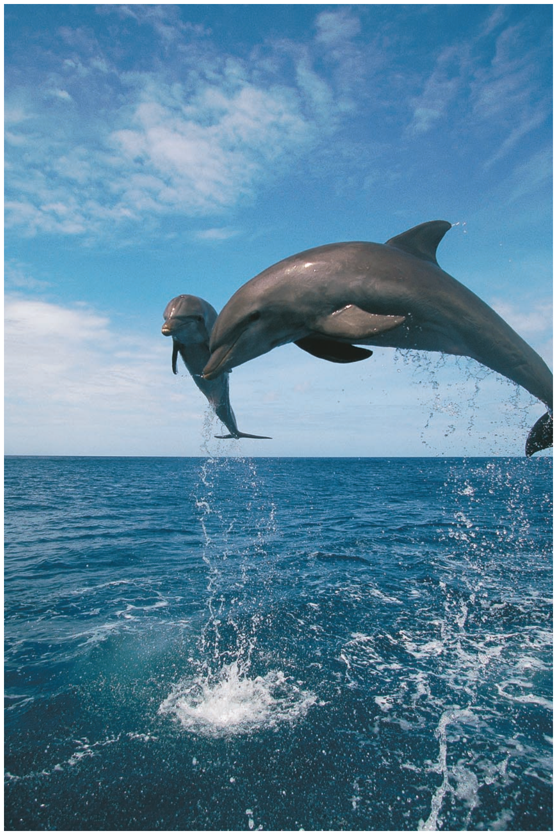


Figure 2.3 Dolphins have been seen jumping as high as 4.9 m from the surface of the water in a behaviour called a "breach."